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Towards a Theory of Irregular Variations in the Length of the Day and Core-Mantle Coupling [and Discussion]

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Towards a theory of irregular variations in the length of the day and core–mantle coupling

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Determinations of fluctuations in the length of the day reveal changes due to the transfer of angular momentum between the Earth's 'solid' mantle and the overlying atmosphere on time scales upwards of a few weeks, as well as the slower but more pronounced 'decade variations' due largely (according to current ideas) to angular momentum transfer between the mantle and the Earth's liquid core. Improvements in techniques for monitoring the Earth's rotation, such as those afforded by recent advances in methods of ranging to artificial satellites and the Moon and of very long baseline interferometry, should therefore lead to results of interest to meteorologists concerned with planetary-scale motions in the atmosphere and to geophysicists concerned with the magnetohydrodynamics of the core and the origin of the main geomagnetic field. The consideration of the stresses at the Earth's surface and at the core–mantle interface that bring about angular momentum exchange between the solid and fluid parts of the Earth raises a number of basic hydrodynamical questions requiring further experimental and theoretical research. In the case of the core, quantitative difficulties encountered by the suggestion that the stresses are electromagnetic in origin led to the idea of topographic coupling associated with hypothetical undulations of the core–mantle interface.

1. Introduction

Earlier in this symposium on methods and applications of ranging to artificial satellites and the Moon and very long baseline interferometry, Professor Lambeck gave an excellent account of fluctuations in the length of the day (l.o.d.) due to the exchange of angular momentum between the atmosphere and solid mantle (see, for example, Lambeck & Cazenave 1974, 1976; Markowitz 1972; Morrison 1975; Munk & MacDonald 1960; O'Hora 1975; Okazaki 1975; Smylie, Clark & Ulrych 1973). These fluctuations amount to no more than about 10^{-3} s and are largely associated with displacements in the mean latitude of the major jet streams in the troposphere, the concomitant vertical transfer of angular momentum at the Earth's surface being due to a slight imbalance between the positive torque on the mantle exerted by the surface westerlies in mid-latitudes and the negative torque exerted by the surface easterlies at low and high latitudes. Monitoring the angular momentum 'budget' of the atmosphere with the aid of improved data on l.o.d. changes will doubtless interest meteorologists concerned with fluctuations of the atmospheric circulation on time scale upwards of a few days. One complicating factor is the unknown extent to which the liquid core (see below) might contribute on the same time scales as the atmosphere. This controversial matter will not be settled until as a result of improvements in both the accuracy and the horizontal and vertical coverage of atmospheric wind data – such as those expected from the forthcoming 'first global experiment' (FGGE) of the global atmospheric research programme (GARP) – the atmospheric contribution can be calculated with sufficient accuracy.

The most pronounced of the observed changes in the l.o.d. are the so-called 'decade variations'.

At several milliseconds in amplitude, these fluctuations are too great to be accounted for in terms of atmospheric (or oceanic) processes and geophysicists generally agree that, in the absence of a reasonable alternative, they must be largely due to angular momentum transfer between the Earth's 'solid' mantle of thickness $\doteqdot$ 2900 km and underlying liquid outer core of thickness $\doteqdot$ 2200 km (the radius of the solid inner core being 1300 km to two significant figures) (see, for example, Jacobs 1975; Marsden & Cameron 1966; Munk & MacDonald 1966; Peale 1973; Rochester 1970, 1973; Runcorn 1970; Stacey 1969). The distortion and displacement of the geomagnetic field pattern at the Earth's surface, including the well-known westward drift of the non-dipole field at some $3 \times 10^{-4}\,\mathrm{m\,s^{-1}}$ (see, for example, Bullard, Freedman, Gellman & Nixon 1950; James 1970; McElhinny & Merrill 1975; Yukutake 1973*a*) can be taken as a direct manifestation of core motions, but the accurate determination of all but the broadest features of these motions from geomagnetic observations is an impossible task (Backus 1968). Nevertheless, the quantitative requirement that the time scale and r.m.s. value of fluctuations in zonal speed of core motions be generally compatible with the amplitude of the decade variations in the length of the day is not particularly restrictive (Munk & MacDonald 1960). It is not surprising to find that attempts to establish a strong association between l.o.d. changes and fluctuations in the westward drift of the geomagnetic field (see, for example, Harwood & Malin 1976; Malin & Clark 1974; Yukutake 1973*b*) have been unsuccessful when it is recognized that the drift cannot be regarded even as a rough direct measure of the zonal motion of material in the upper reaches of the core unless it can be shown that other motions, such as the transverse particle displacements found in magnetohydrodynamic waves, are not involved (Hide 1966).

The principal quantitative difficulties arise when the nature of the horizontal stresses that couple the core to the mantle, across the core–mantle interface, are considered (Munk & MacDonald 1960). These stresses must suffice both quantitatively and qualitatively to account for the fluctuating couple at the core–mantle interface implied by foregoing interpretation of the decade variations in the length of the day. Following Hide (1969) we write

$$F = F_\mathrm{v} + F_\mathrm{e} + F_\mathrm{t} \doteqdot 0.04\,\mathrm{N\,m^{-2}}, \tag{1}$$

where F is the average magnitude of these horizontal stresses, the corresponding magnitude of the unbalanced couple between core and mantle being 10^{19}–$10^{20}\,\mathrm{kg\,m^2\,s^{-2}}$. F_v, F_e and F_t are the contributions to F associated with viscous coupling, electromagnetic coupling and topographic coupling respectively.

If δ is the thickness of the viscous boundary layer at the surface of the core then

$$F_\mathrm{v} = C_\mathrm{v}\,\nu\rho\,\Delta U/\delta, \tag{2}$$

where ρ is the density of the core ($\approx 10^4\,\mathrm{kg\,m^{-3}}$), ν is the coefficient of molecular or eddy kinematic viscosity, depending on whether or not the boundary layer is turbulent, ΔU is a typical magnitude of fluctuations in the azimuthal flow speed of the core material relative to the mantle and C_v is a 'viscous drag coefficient' which on general grounds should be around unity.

The electric currents responsible for the main geomagnetic field leak out of the metallic core into the weakly conducting lower mantle and these, together with currents induced in the lower mantle by fluctuations in the main geomagnetic field, give rise to electromagnetic coupling between the core and mantle. If B_1 is the fluctuating horizontal component of the magnetic field at the core–mantle interface and B_2 the vertical component, and μ denotes the magnetic permeability (not significantly different from $4\pi \times 10^{-7}$. H m^{-1}, that of free space) then the term F_e in equation (1) is given by

$$F_e = C_e B_1 B_2/\mu, \tag{3}$$

where C_e is the 'electromagnetic drag coefficient' which, like C_v, should be around unity.

The third term in equation (1), F_t, arises if irregular topographic features are present on the core–mantle interface and h, a typical value of the vertical dimensions of these features, exceeds the thickness of the viscous boundary layer δ. F_t is related to h by the expression

$$F_t = C_t \Omega \rho \Delta U(h-\delta), \tag{4}$$

where Ω is the angular speed of the Earth's rotation ($\approx 10^{-4}$ rad s^{-1}) and C_t is the 'topographic drag coefficient'. C_t is unlikely to exceed unity but (unlike C_v and C_e) may in some circumstances take on very small values (see § 2 below).

In proposing an explanation of the westward drift of the geomagnetic field, Bullard (see Bullard *et al.* 1950) supposed that viscous coupling between the core and mantle is insignificant and introduced the idea of electromagnetic coupling, theoretical models of which have now been the subject of several studies. The work of Rochester (1960) and Roden (1963) raised fears that electromagnetic coupling might fall short by a factor of ten, but more recent work is rather more optimistic in its conclusions (Roberts 1972*a*, *b*; Rochester 1973; Runcorn 1970 (but see Acheson 1975; Alldredge 1975; Roberts 1974); Watanabe & Yukutake 1975).

2. Topographic coupling

The idea of topographic coupling was put forward several years ago (Hide 1969) when electromagnetic coupling seemed quantitatively unattractive. Uncertainties in the values of the individual quantities on the right-hand side of equation (4) make it impossible to estimate F_t accurately at the present time, but it is instructive to consider the possibly conservative case when $\Delta U \approx 10^{-4}$ m s^{-1} – about a third of the average westward drift of the non-dipole component of the geomagnetic field over the past century or so – and ν has the molecular value 10^{-5} m^2 s^{-1} suggested by Gans (1972), 100 times greater than the lowest value suggested in the literature (10^{-7} m^2 s^{-1}) but only 10^{-6} of the upper limit suggested by Toomre (1974) (10 m^2 s^{-1}).

We consider first the thickness of the boundary layer, an expression for which is

$$\delta = (\nu/\Omega)^{\frac{1}{2}} f(\sigma B_2^2/\rho\Omega), \tag{5}$$

where σ is the electrical conductivity of the core and $f(x) = \sec(\frac{1}{2}\cot^{-1}x)/(1+x^2)^{\frac{1}{4}}$. Taking for the core $\sigma \approx 5 \times 10^{-5}\,\Omega^{-1}$ m^{-1}, $B_2 \approx 5 \times 10^{-4}$ Wb m^{-2}, $\rho \approx 10^4$ kg m^{-3} and $\Omega \approx 10^{-4}$ rad s^{-1}, we find $\sigma B_2^2/\rho\Omega \approx 10^{-1}$ and $f \doteqdot 1$, so that

$$\delta \doteqdot (\nu/\Omega)^{\frac{1}{2}} \approx 10^2 \nu^{\frac{1}{2}}\,(\mathrm{m}) = 0.3\,(\mathrm{m}) \quad \text{when} \quad \nu \approx 10^{-5}\,\mathrm{m^2\,s^{-1}}$$

(the corresponding values for $\nu \approx 10^{-7}$ m^2 s^{-1} and 10 m^2 s^{-1} being 0.03 and 300 m respectively). The corresponding measure $\delta \Delta U/\nu$ of fluctuations in the Reynolds number of the boundary layer flow is 3 (or 30 if $\nu \approx 10^{-7}$ m^2 s^{-1} and 3×10^{-3} if $\nu \approx 10$ m^2 s^{-1}), indicating that the flow should not be turbulent and justifying the assumption that ν can be taken as the coefficient of molecular viscosity without fear of serious error. The corresponding value of F_v is 3×10^{-5} N m^{-2} (when $\nu \approx 10^{-5}$ m^2 s^{-1}) which is less than F by a factor of 10^3; even if it were reasonable to suppose that ν is as high as 10 m^2 s^{-1}, the corresponding value of F_v would still be less than F.

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The shape of the core–mantle interface is unlikely to be a figure of revolution to an accuracy of 0.3 m (i.e. less than 1 part in 10^7), so we can safely assume that $h \gg \delta$ and write

$$F_t = C_t \Omega \rho \Delta U h \tag{6}$$

in place of equation (4). Taking $F \approx F_t$ and $\Delta U \approx 10^{-4}\,\mathrm{m\,s^{-1}}$, we find from equations (1) and (4) that topographic coupling would be adequate if

$$h \gtrsim 400 C_t^{-1}\,(\mathrm{m}) \tag{7}$$

but not otherwise, showing that if, for example, $C_t \doteqdot 1$ then topography with h as small as 1 km would easily suffice. Owing to the complicated dependence of C_t on h, however, equations (6) and (7) cannot be taken as indicating that *high* topography necessarily implies *strong* topographic coupling.

This dependence has not yet been fully investigated, but quite general arguments suffice to show (see below) that C_t should *decrease* with increasing h when h exceeds a certain critical value h_* (see equations (8)–(10)), dropping to values typically very much smaller than those attained when $h < h_*$. The determination of h_* and the investigation of the detailed dependence of C_t on h and other parameters, particularly over the range $h < h_*$, are matters of considerable importance in geophysical fluid dynamics. A detailed discussion of this problem of flow over and around topography in a rapidly rotating fluid, with its various ramifications, would be out of place here, but some of the main ideas, results and uncertainties can be indicated quite simply. Consider a rotating liquid in a container of dimensions D parallel to the axis of rotation and R transverse to the axis of rotation, with topographic irregularities on the bounding surfaces of typical axial dimensions h. We denote by U a typical transverse relative flow speed, L a typical transverse length scale of the pattern of flow, τ a time-scale characteristic of changes in the flow pattern, B a typical magnetic field strength, $V = B/(\mu\rho)^{\frac{1}{2}}$ being the corresponding Alfvén speed, and $\rho N^2/g$ a typical value of the vertical gradient of potential density, g being the acceleration of gravity and N the so-called Brunt–Väisälä frequency, where $N^2 > 0$ corresponds to stable density stratification on the average and $N^2 < 0$ to unstable density stratification. Attention will be confined to the case of a fluid, such as the Earth's liquid core, which rotates so rapidly that $U \ll L\Omega$, $\Omega\tau \gg 1$ and $\delta \ll D \ll L^2\Omega/U$, so that the Coriolis term in the equation of motion is very much bigger than the other acceleration terms and the viscous term.

We consider first the case when (a) magnetic and buoyancy effects can be neglected (notably when $V/U \ll 1$ and $|N| \ll L\Omega/D$) and (b) the non-linear inertial term in the equation of motion, though small in comparison with the Coriolis term, is bigger than the viscous term and the time-varying term (notably when $U/L\Omega \gg \max(\nu/L^2\Omega, 1/\Omega\tau)$. Detailed mathematical models are not easy to construct and analyse owing to the essential nonlinearity of the problem, but general physical reasoning based on the vorticity equation and supported by experimental studies indicate that when $h > h_*$, where

$$h_* = \alpha D U / L\Omega, \tag{8}$$

α being a constant $\gtrsim 1$, the flow is virtually two-dimensional in planes perpendicular to the axis of rotation, with 'Taylor columns' present over topographic features of the bounding surfaces (see Hide 1961; Hide & Ibbetson 1966). So far as the distortion of the flow pattern is concerned, when $h > h_*$ topography interacts strongly with the moving fluid, but, paradoxically, the concomitant force acting on a typical topographic feature has a very small 'drag' component compared with the 'lift' component acting at right angles to the mean flow, that is to say $C_t \ll 1$.

The lift component is almost exactly $2\Omega\rho U$ times the volume of the feature, corresponding to a 'lift coefficient' equal to unity.† If for the core of the Earth we take

$$L \approx 10^6\,\mathrm{m} \quad \text{and} \quad U \approx 3\times10^{-4}\,\mathrm{m\,s^{-1}}$$

we find $U/L\Omega \approx 3\times10^{-6}$ and $\alpha DU/L\Omega \approx 10\alpha$ (since $D \approx 3\times10^6\,\mathrm{m}$). Experiments with systems having $h \doteqdot L$ (Mason 1975) yield for α values around 20 and if, in the absence of reliable information for the case $h \ll L$, we take $\alpha \approx 20$ in the present calculation, we find $h_* \approx 200\,\mathrm{m}$. This implies that (*a*) in the absence of effects due to magnetic fields and/or buoyancy forces and to time variations in the pattern of flow, topography with h in excess of about 200 m would strongly influence the pattern of core motions, but (*b*) owing to the very low value of C_t when $h > h_*$, there would then be no significant topographic coupling of the core to the mantle!

When h is less than h_*, the topographically induced disturbance to the flow is three-dimensional and the net force acting on a topographic feature can have a substantial drag component, in contrast to the case when $h \gg h_*$. There is evidence (see, for example, Mason 1975; Stewartson 1954; Taylor 1921) that values of C_t of around unity apply when the motion is three-dimensional provided that h is comparable with L, but the situation when $h \ll L$ is still confused and requires further work. It is conceivable that in the case when $h \ll L$, the value of C_t attains values of around unity only when time variations in the flow pattern are not negligible (i.e. when $\tau \ngtr L/U$ and $\alpha D/\Omega \min(\tau, L/U)$ replaces the right-hand side of equation (8)), but this remains to be investigated more fully. Time variations in core motions may not be negligible and values of C_t of around unity may therefore be appropriate, in virtue of such variations, provided, of course, that $h < h_*$.

Density inhomogeneities and magnetic fields should affect topographic coupling largely through their influence on the value on h_*, the critical value of h above which C_t falls off with increasing h (see equation (4)). So far as density inhomogeneities are concerned, if the vertical density gradient is top heavy (i.e. $N^2 < 0$) then convective overturning would keep $|N^2|$ very close to zero. On the other hand, when the vertical density gradient is bottom heavy (i.e. $N^2 > 0$) h_* can be obtained from equation (8) by replacing D with $L\Omega/N$ when the former exceeds the latter; thus

$$h_* = \alpha(U/L\Omega)\min(D, L\Omega/N), \tag{9}$$

giving $h_* = \alpha U/N$ when $D > L\Omega/N$. The value of N is not known for the core (see Cox & Cain 1972; Jacobs 1975) but equation (9) shows that density inhomogeneities are unlikely to raise the value of h_* and thus enhance topographic coupling.

The magnetic field in the core will be much more important, but without specific theoretical models of the dependence of h_* on V its effects can only be conjectured. Very rough physical arguments that h_* can be obtained from equation (8) by regarding D as an upper limit to the scale of axial variations in the flow pattern and replacing U with $\max(U, V^2/U)$ (see Acheson & Hide 1973, especially, equation (3.17), of Anufriyev & Braginsky 1975), give

$$h_* \leqslant \alpha D \max(U/L\Omega,\ V^2/UL\Omega). \tag{10}$$

For the core, V could be greater than U but is unlikely to exceed $(L\Omega U)^{\frac{1}{2}}$, the value ($\approx 10^{-1}\,\mathrm{m\,s^{-1}}$) corresponding to approximate equality in the magnitude of Coriolis and Lorentz

† Poincaré's theorem (see Toomre 1966; Malkus 1971) that fluid in a rotating precessing spheroidal container moves as a solid body with the container when the angle of precession is small applies, presumably, to the case when the axisymmetric equatorial bulge exceeds a critical value analogous to h_* (see equation (8)). It is a lift force that then keeps the fluid moving with the container. In theoretical work which effectively takes $U/L\Omega$ equal to zero, such as Poincaré's, even infinitesimal topography gives rise to Taylor columns and to concomitant forces with a dominant 'lift' component.

forces. The comparison of equations (8) and (10) suggests that the presence of a magnetic field might greatly increase h_*, the critical value of h below which essentially three-dimensional motions occur, and a drag coefficient C_t of around unity might be expected; indeed, in the case when $V \approx (L\Omega U/\alpha)^{\frac{1}{2}}$ we have $h_* \lesssim D$, implying highly three-dimensional motions and comparatively large values of C_t.

The foregoing discussion shows what further work is needed before we can say with confidence whether or not it is justified to take $C_t \doteqdot 1$ in equation (7). The careful evaluation of the evidently crucial if subtle rôle played by the magnetic field will require the analysis of theoretical models in which spatial variations in the basic (undisturbed) magnetic field and possibly time variations in the basic flow are taken into account.

3. Concluding remarks

Bumps of around a kilometre in vertical dimensions h and a thousand kilometres or so in horizontal dimensions L have also been invoked in discussions of processes that might influence flow patterns in the core and thus affect the shape of the geomagnetic field and its characteristic time variations (Hide 1966, 1967; Creer 1975). Owing to the large density contrast at the core–mantle interface, such bumps would make a significant (though not dominant) contribution to the low degree harmonics of the gravitational field observed at the surface of the Earth (Cook 1963; Hide & Horai 1968; Kaula 1969). So, it has been argued, if bumps also distort the magnetic field then the two fields might be expected to show a significant degree of correlation when one field is displaced relative to the other (Hide 1967; see also Moffatt & Dillon 1976), and this indeed was subsequently found to be the case (Hide & Malin 1970, 1971 *a*, *b*, *c*, 1972).

The discovery that the Earth's gravitational and magnetic fields are spatially correlated cannot, of course, be taken as unequivocal evidence in favour of bumps, since there are other ways, albeit less attractive, in which the correlation might be explained. Seismology is not yet able, unfortunately, to provide evidence that might settle whether or not the core–mantle interface is bumpy on the postulated scale, in spite of significant progress made in recent years with the seismological investigation of horizontal variations in the properties of the lower mantle (Bolt 1972; Bufe & Carder 1972; Haddon 1972; Jordan 1975; Phinney 1972; Wright & Lyons 1975). Methods capable of detecting and mapping topography on vertical scales as low as 10^3 m have yet to be developed.

Further interesting questions arise when processes that might possibly deform the core–mantle interface are considered. Convection in the mantle (see, for example, McKenzie, Roberts & Weiss (1974), Robinson (1974), Runcorn (1962)), if it occurs at great depths, is an obvious candidate (Hide 1967, 1970), but there have been other proposals (see Anderson 1972; Schloessin 1974) such as mechanical, chemical or electrochemical erosion of the interface and constitutional supercooling. Investigations of these and other suggestions will be handicapped by the lack of direct observations in this complex business of unravelling the mysteries of the Earth's deep interior, but if they follow the example outlined in this paper they will produce valuable interplay between theory and experiment in the interpretation of a variety of surface observations, including the new data to be derived from ranging to artificial satellites and the Moon and very long baseline interferometry.

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Discussion

J. A. Weightman (*Geodetic Office, Elmwood Avenue, Feltham, Middlesex*). If there are bumps on the core–mantle interface, why not also bumps at the interface between the liquid core and the solid inner core? Could they produce similar magnetic and gravimetric effects and if so would they need to be larger or smaller than the others?

R. Hide. Motions in the liquid outer core will, of course, be influenced by the shape of both bounding surfaces, but undulations of the *lower* boundary, at the interface with the solid inner core, would have to exceed 10^2 km or more in vertical scale in order to contribute significantly to the regional gravitational field at the Earth's surface.